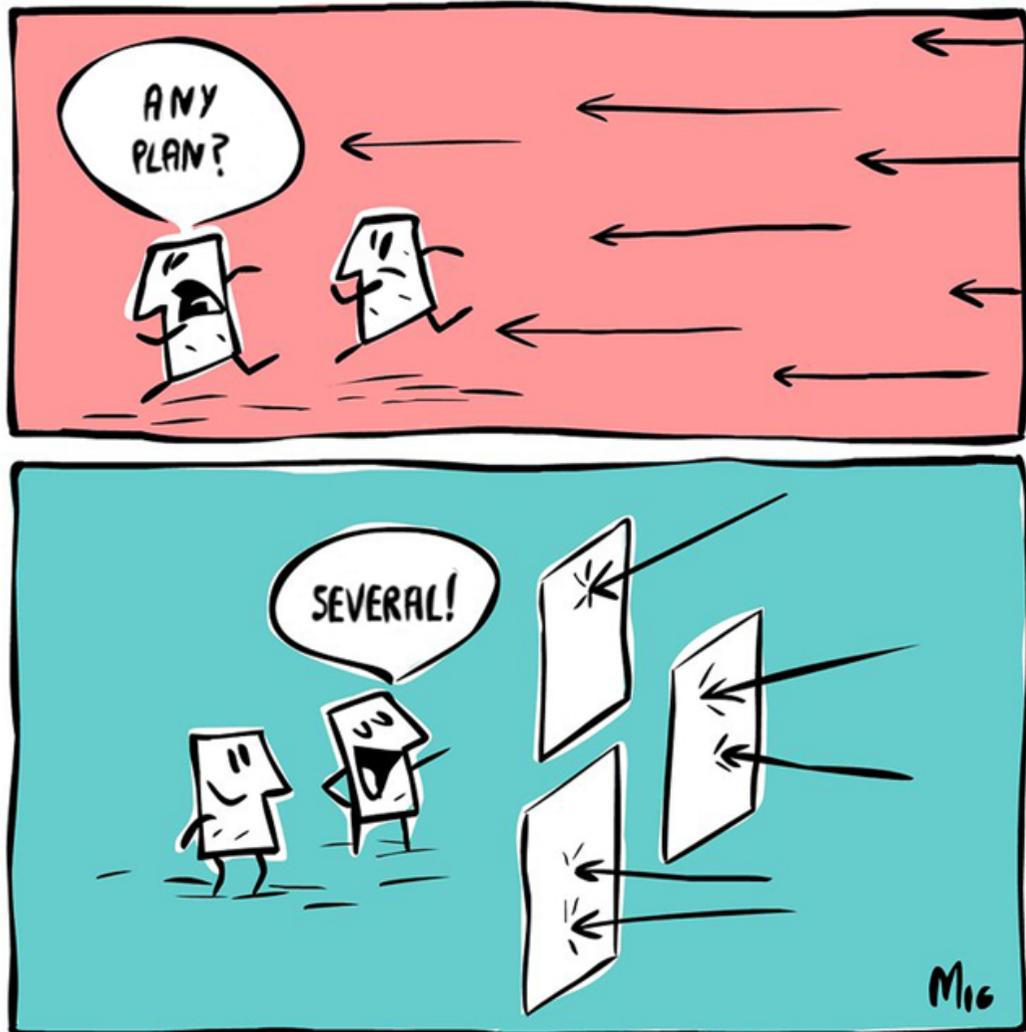


HUMOUR

IN MATHEMATICS TEACHING

Tasks for the classroom



Luís Menezes. Helena Gomes. António Ribeiro. Ana Patrícia Martins. Pablo Flores.
Floriano Viseu. Ana M. Oliveira. Isabel A. Matos. João P. Balula. Véronique Delplancq

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(language revision)

2019

Title: Humour in mathematics teaching: tasks for the classroom

Original title: Humor no ensino da matemática: tarefas para a sala de aula

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Language revision: Ana Maria Costa and Susana Amante

ISBN: 978-989-54036-6-0

Date: 2019

Place of publication: Viseu

Publisher: Litoprint and Higher School of Education of Viseu (Portugal)

Edition: Non-commercial edition

Acknowledgements: This work is financed by national funds through FCT – Science and Technology Foundation, I.P., under the project UID/Multi/04016/2016. Furthermore, we would like to thank the Polytechnic Institute of Viseu and CI&DETS for their support.



We also thank each one of the authors of the cartoons, identified throughout the text, the opportunity to create mathematical tasks of a humorous nature for the classroom, based on their work.

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Introduction

This book, *Humour in Mathematics Teaching: tasks for the classroom*, is a corollary of HUMAT Research Project (*Humour in Mathematics Teaching*), developed by the Higher School of Education of Viseu (Portugal), in partnership with the University of Minho (Portugal), the University of Granada (Spain) and the University of Mendoza (Argentina), with the support of the Polytechnic Institute of Viseu (Portugal) and of the CI&DETS.

The project rests upon two fundamental assumptions. On the one hand, it assumes that humour plays a crucial role in creating a learning environment that can boost the students' motivation and encourage them to learn mathematics. On the other hand, it assumes that the understanding of humour and the learning of Mathematics are two activities that require a good reasoning capacity. It also recognises that the inquiry-based mathematics teaching, grounded on the work of the students leading to the completion of challenging mathematical tasks, provides a high potential for learning. All those considerations led to the birth of this book that offers mathematical tasks with a humorous twist and whose main goal is to fulfil these two functions: to persuade students to learn and make them think about humorous situations that involve mathematical concepts.

To achieve this goal, the book is organised into two sections. The first is devoted to a brief introduction to the concept of humour and the other concepts that are associated to it approaches the concept initially in a broader sense, then focusing on the teaching of Mathematics. The largest section of the book, the second, is devoted to the presentation of mathematical tasks that are based on panels or comic strips involving humorous and witty content.

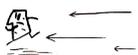
The tasks suggested were planned for an exploratory-type teaching of mathematics and therefore they generally have an open nature. Almost invariably, we start by asking the student to describe the situation presented and to say whether or not they consider it funny. After this first request, we present others of a more specific nature.

The selected tasks focus on mathematical topics that are taught in basic education. Our proposals can be used in two different circumstances. Some were designed to introduce new topics and others to consolidate previously learned knowledge. Tasks can be used and explored at different school levels, as long as we adapt the questions' instructions or the way we guide the students' work.

The tasks are different according to the kind of reaction they are meant to trigger in students. Some tasks value affective/emotional aspects as they seek to seize the students' attention and keep them



engaged in their learning, thinking about an unexpected situation presented. Other tasks are focused on the development of cognitive and intellectual aspects. With those tasks, the students' work goes far beyond the mere focus on learning. It represents challenges that arise from the cognitive ruptures that were conveyed by the proposed situations/tasks. For each task, we present a brief description of the situation and then we come up with didactic considerations about the several questions involved.



Let's talk about humour in Mathematics teaching

Setting the scene for humour

Humour is an integral part of human activity and can occur in many ways (jokes, puns, imitations, illustrations, cartoons), in different situations (formal, informal), in different ways (spontaneous, previously prepared) and directed to different target groups or institutions (social institutions, politicians, athletes). Humour can be seen as an existential attitude that allows people to accept or to better understand reality, removing stress-generating aspects and creating distension environments which will ideally elicit responses such as laughter and joy (Adão, 2008; Banas, Dunbar, Rodriguez & Liu, 2011; Martin, 2007; Meyer, 2015).

The concept of humour has evolved historically and has been studied within many scientific fields, like psychology, linguistics, philosophy and sociology. Banas et al. (2011) argue that "humour involves the communication of multiple incongruous meanings that are amusing in some manner" (p. 117). Humour is thus a form of communication that abundantly uses ambiguity and polysemy, balancing cognitive and affective elements to make someone laugh (Guitart, 2012; Martin, 2007; Meyer, 2015).

How humour works. Humour has been studied from several theoretical perspectives which offer different visions about how humour happens. Of all the different theories that seek to explain humour, we will point out the following: the Superiority theory, the Incongruity theory and the Relief theory (Adão, 2008; Adão & Oliveira, 2011; Bergson, 2011; Martins, 2015). In the first theory, humour results from a certain sense of superiority over someone or something, which may lead to the disparagement of another person's behaviour. This mechanism is very much present in British humour and in political satire also (Adão, 2008; Martins, 2015). Incongruity is a very common mechanism in humour, in which two conflicting and incoherent ideas or events are combined in an unexpected and funny way. Humour happens when the reader resolves this apparent incongruity (Adão, 2008; Martins, 2015). The third theory regards humour as a release of tension felt in a given moment and context. That tension is suddenly shattered by an unexpected and hilarious event (Adão, 2008; Martins, 2015).

Due to the specificity of humour, the understanding of a humorous message depends on countless factors, such as the individual's language and personality. In addition, the ability to assess humour will also change as people grow up from their early years of life to adulthood.

Sense of humour. Sense of humour is a way of looking at the world, a style, a way of self-protection and a strategy used to overcome difficulties. It is also a form of attack, a certain kind of behaviour people adopt to show kindness, to help someone in difficult situations and to gain the trust and appreciation of others (Thorson & Powell, 1993). Apart from the individual structure of the sense of humour that provides people with a wide range of humorous instruments, the way in which each one of us uses humour can change depending on the individual's personality, on that person's mood and feelings at a given moment, on the situation that the person is experiencing and on the importance given to such situation (Adão, 2008). According to Ruch and Hehl (1983), the sense of humour depends upon our acknowledgement of oneself as an individual with humour and of the others' humour as well, humour appreciation, laughter and finally the ability to resort to humour as a coping mechanism.

All the personal achievements and every individual failure are very important to a person's self-image because they condition the level of motivation which is needed to enable the development of the sense of humour. A person who feels that his/her humorous interventions are usually enjoyed by others tend to think that he/she is a funny person and has no reason whatsoever to restrain himself/herself from resorting to humour.

On the other hand, those who often have trouble finding enjoyment in humour in different contextual or verbal situations are more likely to feel reluctant to use or value humour (Coulson & Kutas, 2001). The gap that exists between the attitude developed towards other people's humour and humour itself is very tenuous, since each person's subjectivity has an impact on that person's tastes and on his/her perspective of the different types of humour. In short, sense of humour is not a unified trait, but instead has a very close relationship with behaviour, cognition and personality.

Humour in Mathematics teaching and learning

Nowadays, humour is a highly socially valued trait and can be found in countless contexts of human activity. Humour is strongly present in entertainment activities like theatre, cinema, radio, television, internet and literature since there is no doubt that it plays a crucial role in the well-being and in the release of the tension caused by the problems and stress issues of our everyday life (Adão, 2008; Celso, Ebener & Burkhead, 2003). In the business world, humour is used as a productivity enhancer and also as a negotiation facilitator (Gockel, 2017). And what about education? And especially what about mathematical education? Is there any way we can use humour in classrooms with educational and instructional value?

Recent papers based on the review of the literature dealing with the use of humour in education show that researchers are really interested in the topic, mainly those who have conducted their studies during the late twentieth century (Banas et al., 2011; Martin, 2007). Many of these studies show that humour has a positive effect on creating good learning environments and on increasing students' attention span. Other studies also point out the use of humour to relieve tension when teaching topics are usually difficult for students and can, therefore, cause anxiety (Banas et al, 2011).

Most of the studies reviewed by Banas et al. (2011) reveal that the use of humour is essentially a communicative strategy to support teachers' oral interventions, and that this humour is more or less focused on the topics to be taught. The study conducted in Portugal, with about 600 teachers who teach mathematics from primary school to higher education (Menezes, Viseu, Ribeiro & Flores, 2017) shows that about 90% of the teachers surveyed admit they use humour in their classes and over half of them confirm that they do it on a regular basis. When asked about the way they use humour, they present examples that demonstrate that it happens mainly during the teacher's oral interventions and that it is focused on mathematical topics.

In the following examples, teachers who have participated in our study have used questions, riddles and expressions that contain puns involving mathematical concepts in their dialogue with the class in order to hold students' attention and to help them retain the information: "When, in geometry, we talk about the radius of a circle (*"raio" in Portuguese*) I usually ask: who knows what's worse than being struck by lightning (*"raio" again in Portuguese*)?" (p. 63); "I resort to humour, particularly to encourage them to look for Mathematical Proof" (p. 64). This teacher explains: "an expression I use quite often is something like: "if you cannot find counterexamples to refute the conjecture, you have two choices: you're either dumb or it is a mathematical impossibility" (p. 64). Consequently, the teacher says that "from that moment on, we have amusing conversations in which the students, trying not to look "dumb", strive to take their analysis of the validity of the conjecture a few steps further" (p. 64).

Humour, with its subversive side, seeks to "disarm" students and remove them from a possible defensive attitude towards mathematics, by triggering sensations of well-being when students finally understand how funny the situation is. In the classroom, the humorous situations presented by the teacher can achieve two fundamental functions: an affective/emotive function and an intellectual/cognitive function (Banas et al., 2011; Guitart, 2012; Meyer, 2015). When the humorous situation is not directly related to the mathematical topic you want to teach, the affective/emotive function stands out, as the teacher does his /her best to create a good learning environment. On the other hand, when the humorous situation is directly related to the mathematical topic, the intellectual/cognitive function takes the lead. In this case, humour plays an important role in the learning

of the topic that is being taught, either because it is present in some of the tasks, because it helps understand the concept or simply because it facilitates the memorisation of some name or idea.



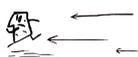
How the book is organised

This book aims to show how we can use humour in maths classrooms, and relies upon two deliberations/resolutions. On the one hand, it emphasises graphic humour over the teachers' verbal humour. On the other hand, this kind of humour will be used to have a good impact on the learning of mathematics based on a balanced combination of the emotional and cognitive functions.

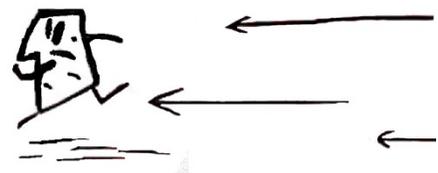
The tasks are open-oriented and humour will be used as a mathematical challenge for the students. Therefore, the tasks were designed to be used in an exploratory teaching environment in which the mathematical activities offered to the students are meant to bring about learning.

Typically, an inquiry-based teaching class of mathematics follows three or four phases (Navarro, Oliveira & Menezes, 2014; Stein, Engle, Smith & Hughes, 2008). Stein et al. (2008) suggest a three-stage process class: launching, exploring and discussing the task (with a final summary). Other authors, like Navarro, Oliveira and Menezes (2014) split the last phase: for them, there is a difference between collective discussion, in which the pupils are the main protagonists, and the "systematization of learning" phase where the teacher regains the leading role, by involving the students in the internalisation of the mathematical knowledge, which results from a reflective abstraction based on the task performed and on others performed before and that allows them to establish connections with other mathematical topics.

In this book, 15 mathematical tasks are presented, and in most cases, humour arises from situations of incongruity involving the use of mathematical concepts. This is what happens, for example, in the tasks "Attack!", "When number two is not that big a deal..." and "A smaller map." In other tasks, humour relies on puns that involve natural and mathematical language, like in the task called "All right!", and ambiguities of language, like in the task "Degrees and degrees". Other tasks rely on internal mathematical problems, both in relation to the conclusions that students were able to draw from the task, like in the task called "Irregular regularity", and from the language that is used, in the task "Right or Wrong?"



Mathematical Tasks



Attack!



Hagar, the Horrible, Chris Browne

1. Describe the situation depicted in the comic strip. What could have been the intention of the protagonist? What strategy did he use? Do you consider this situation funny?
2. How many numbers does Lucky Eddie have to enumerate before he attacks the enemy? What kind of numbers did he use? And which representations occur?
3. How could he reduce the waiting time? What if he wanted to delay the attack even further?
4. Imagine that Lucky Eddie keeps on counting until he gets to $9\frac{7}{8}$. What strategy could he use to delay the attack even further?

¹ This task is part of the set of material developed to support the implementation of the Mathematics Educational Programme for Basic Education (Menezes, Rodrigues, Gomes & Tavares, 2009).

Instructions on how to explore the task

Description of the situation

This mathematical task is based on the comic strip “Hagar the Horrible”, created by the American cartoonist Chris Browne. The strip is formed by two panels. On the first panel, Hagar and his companions are under attack and, apparently, a counterattack is being prepared. Hagar asks Lucky Eddie to count until 10, and then they will attack. In the second panel, we are surprised with the hilarious strategy adopted by Lucky Eddie who, unexpectedly, uses fractions to delay the countdown and thus delay the attack.

The task

With this task, it is possible to get students to compare and order rational numbers represented in different ways and also to locate and place on a number line a given non-negative rational number.

The task, based on the strip, initially aims at leading students to understand what could possibly have motivated this strange way of "counting", in other words, we want to clarify what the protagonist's intention was. This mathematical understanding allows students to appreciate the humour present in the strip.

With the second question, students are meant to indicate how many numbers would have to be said until they could reach 10 and also to explore different forms of representation.

In the third question, students are expected to use their knowledge of rational numbers and understand that the denominator of the fraction is related to the "quantity" of numbers that have to be said until the attack is initiated, i.e. using unit fractions $\frac{1}{n}, n \in \mathbb{N}$ (as in the strip), the increment of n increases the waiting time and the decrease of n would have the opposite effect. The "quantity" of numbers that have to be said is given by the expression $10n$. It would also be interesting to discuss what happens when we use non-unit fractions.

In the fourth question, students are challenged to find non-negative rational numbers between $9\frac{7}{8}$ and 10. Students can be led to understand that between two rational numbers it is always possible to find another rational number. For that purpose, they should have the opportunity to use different forms of representation, establishing connections between them.

When number 2 is not that big a deal...



1. Describe the situation presented in the comic strip. Do you think the situation is amusing?
2. Assuming that on that day numbering started at 0, how many people might already have been assisted?
3. If this numbering was to continue as suggested in the panel, how many people are to be assisted until they reach number 1? And number 2?

Instructions on how to explore the task

Description of the situation

This mathematical task uses a comic strip from the *Toon Hole*² blog, created by the American graphic illustrator Ryan Kramer.

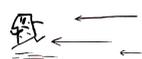
The situation presented is based on a common daily situation: the use of natural numbers to determine the order of a sequence that is usually used to define our place in a waiting queue. The male character is quite satisfied with the number he received (number 2) because he is sure he won't have to wait a long time to be assisted, since reception counters usually start with number 1 (or with 0 at the best). The unexpected twist in this situation is that the last person to be called is not an integer (0.001271), and therefore the time he has to wait until the counter reaches number 2 can be much longer than he was expecting. The fourth panel expresses, hilariously, the surprise, or perhaps the disappointment felt, when he notices that the "counting" is not carried out as usual.

The task

This task points out the use of numbers in everyday life, especially when people are given a call number when waiting in a queue. This situation allows us to discuss how Mathematics is useful and important in our everyday life. The task allows us to work with the extension of \mathbb{N} to \mathbb{Q} and the student is expected to understand that between two natural numbers there are infinite rational numbers. We can also explore the ordering of rational numbers in the decimal representation.

The first question draws the students' attention to the meaning of numbers. At first, we want the student to recognise that natural numbers give an idea of order (ordinal numerals). Unlike rational numbers, natural numbers allow you to know how many numbers there are before a given number (assuming you know when the "counting" started). In order to make the exploration of the task a bit easier, the number of decimal digits was limited to six. Therefore, the situation suggests that the "counting" process will follow the sequence: 0.000000 0.000001; 0.000002; ... However, the task may be presented with more or fewer decimal orders in the representation of the counter's number.

² <http://toonhole.com/>



This reasoning is summed up in the second question when it is clear that the "counting" is done in "millionth digits". This discovery allows students to conclude that 1271 people have already been assisted, and the previous sequence becomes clearer.

The third question is based on the previous assumption: with six decimal digits, the last number before number 1 is 0.999999 and the number that comes right before number 2 is 1.999999.



86499328 of mushrooms
259497984



1. Describe the situation presented in this comic strip. What could have been the little boys' intention? Do you consider the situation amusing?
2. What do you think about the use of fractions in this order? What fractions do you usually use when you want to order different ingredients in the same pizza?
3. How can we know whether or not the three fractions correspond to a whole pizza?

Instructions on how to explore the task

Description of the situation

The mathematical task presented is based on a Fox Trot comic strip, authored by the American cartoonist Bill Amend. In this comic strip, the father, Roger Fox, is ready to pick up a pizza for the "Fox" family. The shop assistant checks the order and says ironically that this pizza has 3 ingredients: cheese, sausage and mushrooms in very unusual portions ($\frac{17}{51}$, $\frac{109}{327}$ and $\frac{86499328}{259497984}$ of cheese, sausage and mushrooms, respectively). The use of these unusual fractions apparently has an impact on the received change, since Roger Fox's pockets are full of pennies.

The task

The task designed from this strip aims to deepen the concept of the fractional representation of rational numbers and its use in real life contexts. In particular, this task allows for study of reducible fractions and how to obtain equivalent fractions (either reducible or irreducible). In the strip, the shop assistant uses the fractions $\frac{17}{51}$, $\frac{109}{327}$ e $\frac{86499328}{259497984}$. This third fraction, which refers to the mushroom portion of pizza, is composed of two even numbers (numerator and denominator of the fraction), which makes it reducible.

In fact, the numerator of the fraction can be decomposed in $2^{14} \times 10559$, which shows that the author wanted to use a number that is 14 times divisible by 2. Since $3 \times 9 = 27$ and $3 \times 100 = 300$, it is easy to conclude that the second fraction is also reducible. In addition, the first fraction presented, and which corresponds to the cheese portion of pizza, has 17 as numerator and 51 as denominator and is therefore equivalent to $\frac{1}{3}$, since $17 \times 3 = 51$. Since the first fraction is equivalent to $\frac{1}{3}$ and the other two are also reducible, looking for an equivalent and irreducible fraction for each of the three fractions becomes more natural.

In the three fractions, the denominator is three times the numerator, which allows us to conclude that each one of them is equivalent to $\frac{1}{3}$. The unusual way in which the change is handed-over (a huge amount of coins) completes and establishes a relationship with the ridiculous situation caused by the request (the large partitioning of the pizza caused by the use of very high denominators).



The first question draws the students' attention to a daily situation, but in which the fractions used to talk about the ingredients of a pizza are not common. The first reading should foster student humour because of the strangeness of the fractions used to represent each portion of the different ingredients that will be used in the pizza.

The second question is used to highlight the importance of rational numbers in real life (in particular their representation in the form of a fraction) and to emphasise the most appropriate representations that have to be used in different situations. This understanding by the students should allow them to see how ridiculous it is to use fractions in this comic strip.

The third question requires students to use the procedures previously described in order to obtain equivalent irreducible fractions. Students will conclude that the sum of the weird fractions used in the pizza order is equivalent to $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$, it will be equivalent to 1 (the whole pizza), in other words.



Fractions... get the hell out of here!!



1. Observe the situation. What do you think of the situation depicted? Do you agree with the characters? Do you consider the situation amusing?
2. In your everyday life, do you use fractions? If you do, in which situations?
3. Overall, which part of the pizza has mozzarella and tuna? And which part has mozzarella and ham?
4. How can we split the pizza in the way suggested by the parrot?

Instructions on how to explore the task

Description of the situation

This mathematical task starts with the exploration of a strip authored by William Raphael Silva, a Brazilian teacher and researcher in Mathematics Education.

Students often ask teachers about the usefulness of the contents they are taught at school. The situation which is caricatured in this strip presents two friends, a monkey and a parrot, who question the usefulness of fractions. Apparently, they both agree that fractions are completely useless in daily life. The third panel, unexpectedly, reveals a hilarious contradiction between the argument presented at the beginning of the story and the extensive use that the two friends make of rational numbers.

The task

With this task, students should understand that they often question the usefulness that certain mathematical concepts may have in their daily life. In the situation presented, the use of fractions is emphasised to draw attention to the meaningful part-whole relationship and students are asked to identify and present other situations that require the use of rational numbers in the form of a fraction. The following questions require the knowledge and understanding of rational numbers represented by a fraction, in particular the operations of addition and division and their corresponding meanings.

Question 4, in particular, enables different resolutions that will help satisfy the Parrot's request, for example:

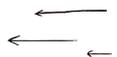
- i) Equal division of the ingredients

$$\frac{3}{16} \text{ of mozzarella} + \frac{3}{16} \text{ of ham} + \frac{2}{16} \text{ of tuna for each half}$$

- ii) Equal division of the total number of slices

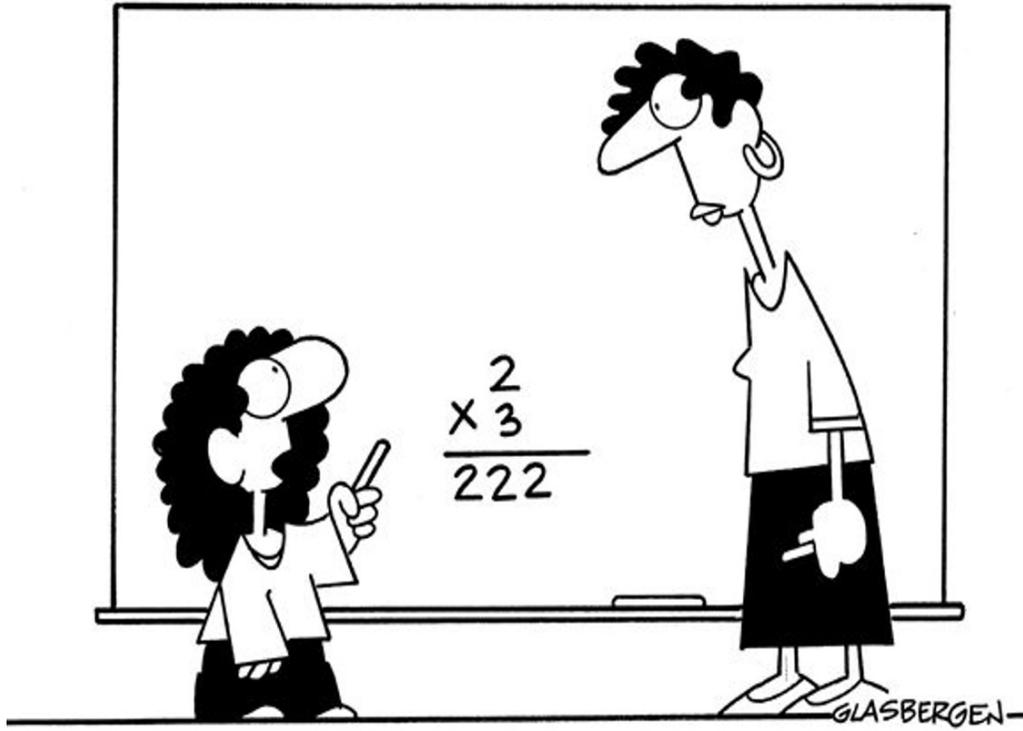
$$\frac{3}{8} \text{ of mozzarella} + \frac{2}{16} \text{ of tuna for one of them and } \frac{3}{8} \text{ of ham} + \frac{2}{16} \text{ of tuna for the other}$$

- iii) Each friend chooses 4 of the 8 slices of pizza containing different ingredients.



Right or Wrong?

© Randy Glasbergen / glasbergen.com



"What do you mean, it's the wrong kind of right?"

1. Describe the situation pictured in the cartoon. Do you think it's funny?
2. Might there be any "truth" in the situation presented? Why?
3. How could you correct the result of the operation without deleting any of the numbers?

Instructions on how to explore the task

Description of the situation

This mathematical task uses an illustration authored by Randy Glasbergen, one of the most popular and published American cartoonists. The author humorously focuses on the habits and customs of our society and on education in particular. In this context, the classroom environment and mathematics teachers are recurring themes, as proved by the illustration offered. Dialogue follows, between the teacher and a student, caused by a "peculiar" use of the multiplication algorithm.

The task

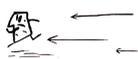
The task designed from that cartoon above is intended to elicit understanding of the of multiplication as a procedure that allows the addition of equal parts. The algorithm pictured in the cartoon suggests that this idea is present in the student's thinking and also in the teacher's intervention when she says it is "a wrong kind of truth". The objection that we can make to the student's conclusion, and that will surely make the class smile, is that our numeral system is not additive and therefore 222 is not the same as $2 + 2 + 2$. This task provides also the possibility of exploring the meanings of multiplier and multiplicand.

In the first question, students are invited to look at the illustration and analyse the teacher's response. The initial strangeness of the multiplication result can lead students to look for any rationality. When they find it, they may understand how funny the situation is.

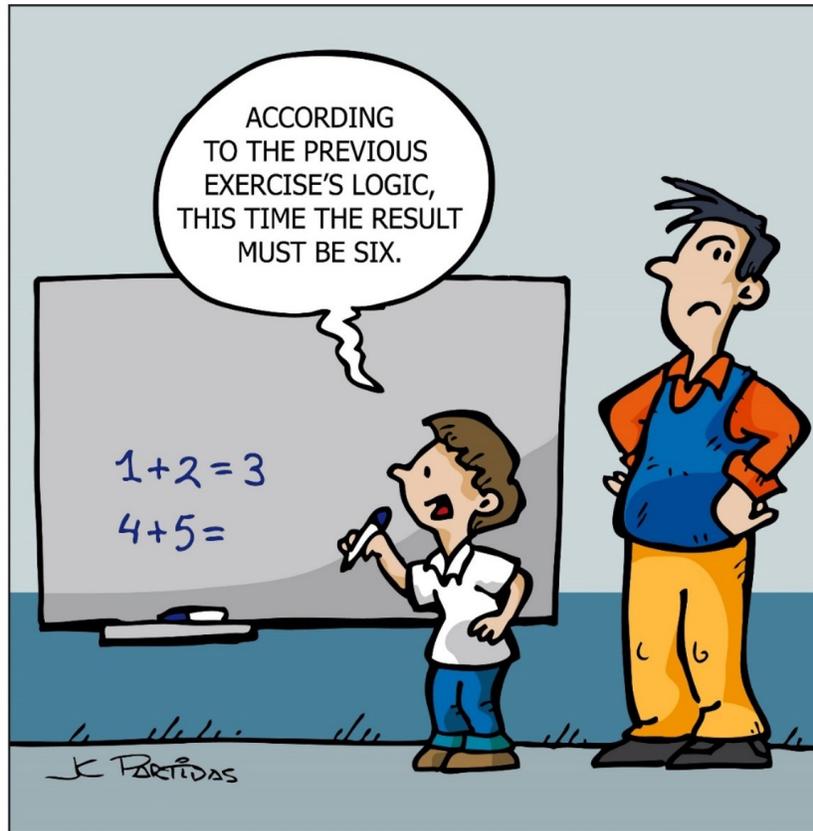
In the second question, the understanding students already have of the decimal numeral system should lead them to interpret the result as a wrong representation of a correct idea, which could be seen as a sneaky way used by the student.

In the third question, since students have already identified the mistake, it is expected that they find ways to correct it. One of the solutions could be to add "+" signs between the three numbers and display the result " $2 + 2 + 2 = 6$ ". Nevertheless, other valid forms of symbolisation can be presented.

In the end, it is important that teachers lead their students to realise that, in the present situation, the use of the multiplication algorithm is totally inappropriate. This was also the cartoonist's intention when he created this cartoon.



Irregular regularity



1. Describe the situation depicted in the cartoon. Do you think it is an amusing situation?
2. Why has the student replied "six"?
3. Find regularities in additions of natural numbers and justify them.

Instructions on how to explore the task

Description of the situation

This mathematical task starts with a comic strip created by Juan Carlos Matches, a Venezuelan designer and author of a blog called *Rechiste*³. He is the author of several strips that use mathematical ideas. In this strip, there are two characters: a teacher and a student. The author describes a normal school situation in which the student has written on the board two numerical equalities, one of which is complete. The student uses the regularity found in the first equality to respond incorrectly to the second. The humour of the situation is caused by the boy's wrong deduction. The teacher is clearly surprised and disappointed with the student's response.

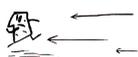
The task

The task allows the teacher to explore concepts like numerical sequence, addition of natural numbers and the regularity in numerical equality. The student is expected to be able to understand and appreciate the humour of the situation and therefore kind of reason underlying the boy's response.

In the question, it is assumed that the student recognises the validity of the first equality and the fact that the response to the second "equality" is incorrect.

In the second question, it is intended that the student understands the kind of reasoning used by the boy, when he answers that the result of $4 + 5$ is 6. After recognising the validity of $1 + 2 = 3$, the boy from the cartoon assumes, wrongly, that the addition of two consecutive natural numbers is equal to the number that comes right after the highest of the two numbers used in the addition. He will then apply this conjecture to the second operation. It is essential that, with this task, students come to understand that, although there is a regularity in the sequence of the numbers, it is no longer valid in the second "equality". Students may also consider the regularity between the parcels, by comparing the second line with the first one (each number from the second line corresponds to a number from the first line to which we add 3) and guessing that the boy could have followed, wrongly, the same reasoning for the sums $(3 + 3)$.

³ <http://elrechiste.blogspot.com.es/>

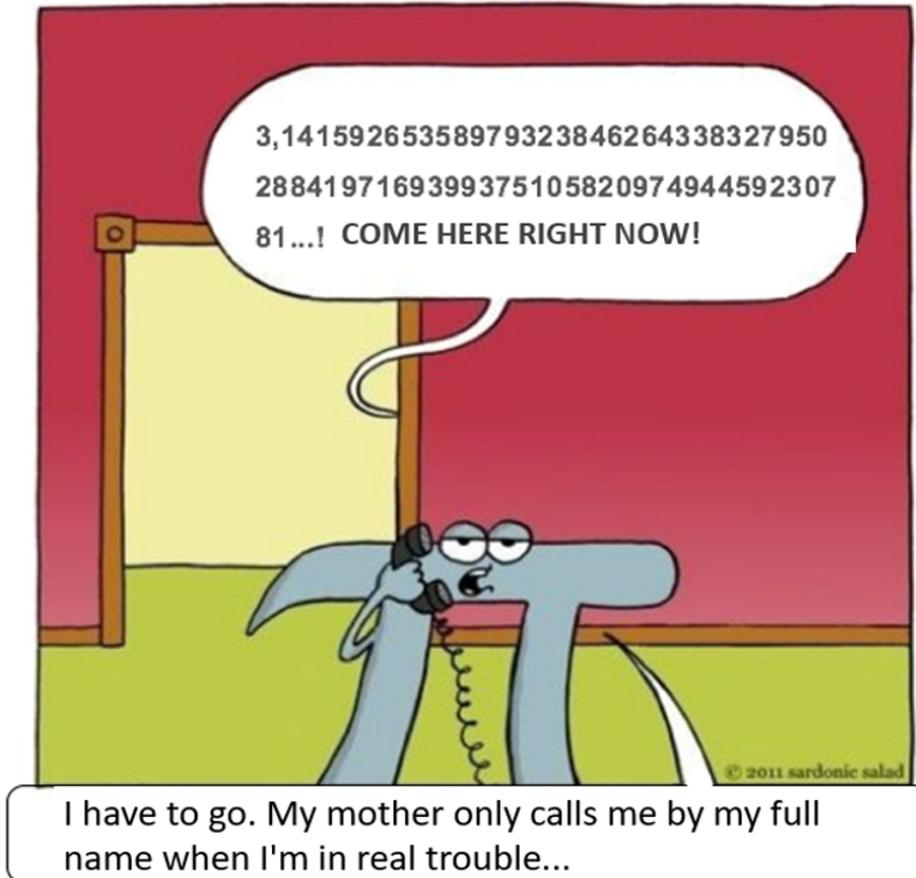


The last request represents an open proposal for the discovery and justification of regularities in addition of natural numbers. It is possible to identify regularities of different natures. The following examples are possible to answer the question:

- i) In a sequence of multiples of natural numbers, the relation $n + 2n = 3n$ is valid for every natural n , the sum of a natural number and its third multiple, $2n$, is equal to its fourth multiple, $3n$, in other words;
- ii) The sum of the first n consecutive natural numbers n is equal to $\frac{n^2+n}{2}$ (a relation known as the *Gauss* conjecture).



Full name, almost full name or incomplete name?



1. Describe the situation depicted in the cartoon. Do you consider it funny?
2. What number are they talking about in the picture? How can you define and classify that number?
3. Do you agree with the statement that the mother is calling him by his “full name”?

Instructions on how to explore the task

Description of the situation

This mathematical task is based on a cartoon taken from a webcomic available on the *Sardonic Salad* blog. In the cartoon, the artist has portrayed a personified number π on the phone, when he is called by his "mother", who, according to him, calls him by his "full name".

The task

The task's goal is to lead students to recognise π as an irrational number, which is represented by an infinite non-repeating decimal. That is the reason why more than 60 decimals followed by an ellipsis are displayed in the cartoon.

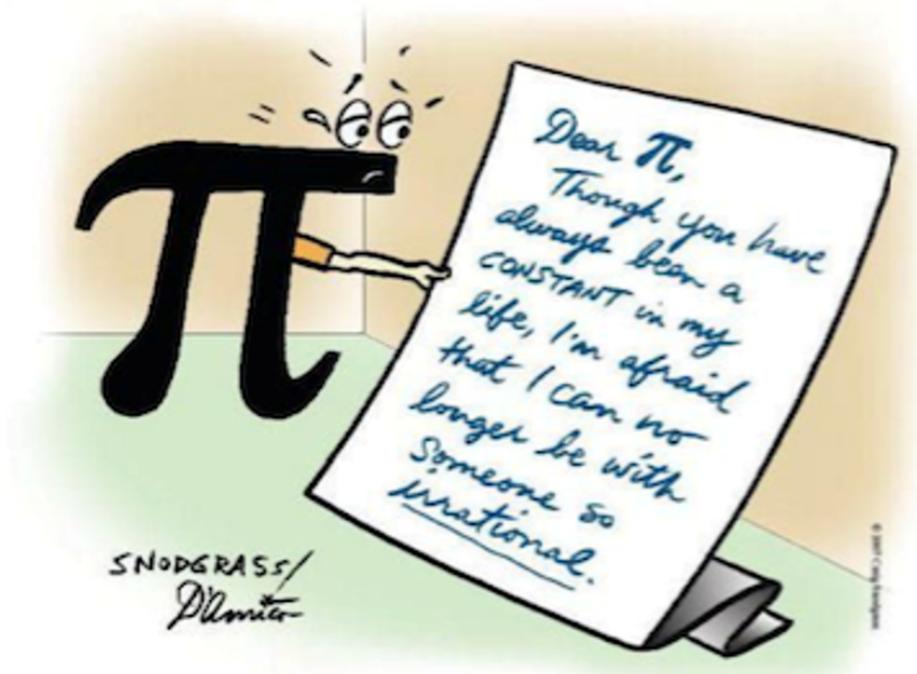
In the first question, students observe the illustration and the unusual form of writing π and explore the fact that it is represented by an infinite non-repeating decimal. The connection between this fact and the way children are sometimes called by their parents (by their full names, when something wrong has happened) can make students laugh.

In the second question, students are asked to identify the number π ; to define it as the ratio of a circle's circumference to its diameter and to classify that number as an irrational number. Students understand that irrational numbers can be represented by infinite non-repeating decimals, as opposed to rational numbers that can be represented by finite or infinite repeating decimals.

In the third question, students must recognise that it is not possible to say π 's "full name" because it is an infinite decimal. That's why the author had to use the ellipsis when writing the character's name. In this context, students can be asked to comment on the title of the task: "Full name, almost full name or incomplete name?".



That's what humour is all about!



1. Describe the situation depicted in the cartoon.
2. Who could have written this letter? How do you justify π 's state of mind?
3. Explain the meaning of the words "constant" and "irrational" in this letter.
4. Do you find the situation funny?

Instructions on how to explore the task

Description of the situation

This mathematical task is based on a cartoon of the American artist Craig Snodgrass. In this illustration, a personified number π gets a letter that makes him sad, since the sender seems to be telling him that their relationship is over. The argument used is that the number π is irrational.

The task

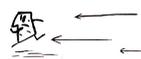
The task presented allows teachers to explore the concepts of numbers and numerical sets and their representations, in particular the concepts of rational, irrational and real numbers. The starting point relies on the polysemic nature of language and on a pun involving the words "constant" and "rational". It is interesting to acknowledge that in mathematics, as it happens in real life, a constant element is an element that does not show any variation. Unlike variables, numbers are constant. π , in particular, is a number that represents the constant ratio of the circumference of a circle to its diameter. The idea of "constant" turns up in the letter because the sender of the message claims that π has been a constant element in her life, as well. It's the double meaning of "irrational" that should give rise to the humour in the situation. On the one hand, an irrational person is usually a person devoid of reasoning or who acts without thinking about the consequences of his/her actions. On the other hand, the number π is an irrational number, since it is a number represented by an infinite non-repeating decimal and that characteristic will ultimately be its downfall and will cause it to lose its sweetheart just because it will be deemed irrational in the non-mathematical, general sense.

In the first question, the students will describe the fact that the number π has received a letter and will identify the contents of the letter.

Based on the content of the letter, it is easy to assume that it was written by a person who maintains a romantic kind of relationship with the irrational number. The emotional state in which we find π is justified by the message delivered by the letter that seems to put an end to their relationship.

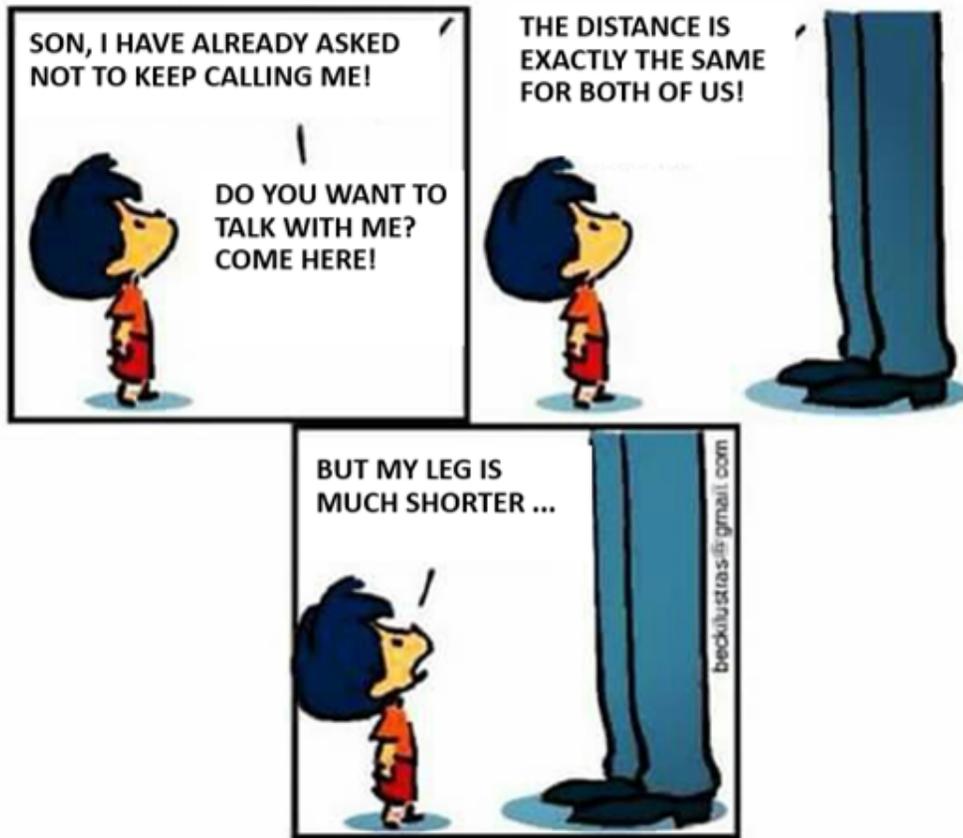
In this second question, students can be asked to indicate possible senders of the message. For example, can the sender be an irrational number? If so, could the letter have had a different content and the story a distinct outcome?

The recognition of the importance that the words "constant" and "irrational" have in the letter allows students to recognise that they are the key to recognise the humour of the situation. In particular, they must realise that being an irrational person in life is something that is not positive but that, in



mathematics, irrational numbers exist, that there are a lot of them and they do not have any "negative trait" (as, in fact, none of the other numbers do). The humour of the situation arises when students fully acknowledge that the message deliberately relies on word play with regard to the use of the word "irrational". In this context, it will certainly be interesting to revise or introduce the concept of irrational numbers and show that it is not possible to express π as $\frac{a}{b}$, with $a, b \in \mathbb{Z}$ and $b \neq 0$ or, in other words, that we can't express π as a ratio between two integers a and b , and $b \neq 0$, as it happens with any rational number.

Equal, albeit different!



1. Describe the situation depicted. Do you find it amusing?
2. Is the boy right?
3. Do father and son have to undertake the same number of steps to reach each other?
What are the differences?

Instructions on how to explore the task

Description of the situation

This mathematical task will start with the analysis of a comic strip, created by Alexandre Beck, a Brazilian journalist and publicist, in which there is a dialogue between a son and his father. The father asks his son to come closer any time he feels like talking to him. He argues that the distance between them is the same for both. The child, in turn, argues that his legs are shorter than his father's.

The task

The task built from this strip deals with the concepts of distance between two points (the measurement of the length of the line segment that connects them). In this case, the length is equal for both but its measurement can be different depending on the unit of measurement that is used. In this situation, the father and the son's steps appear to be the possible units of measurement.

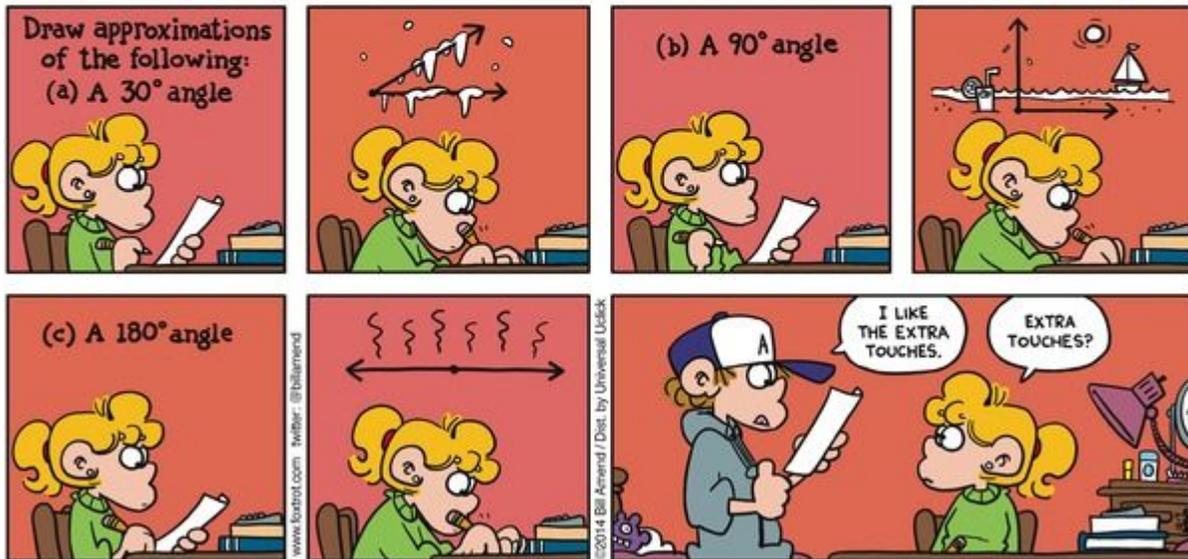
The first question intends to get students to appreciate the strip, considering the arguments presented and the pictures in which they appear.

In the second question, students are asked to assess the arguments presented regarding the distance and the steps that each of the characters will have to take before reaching the other.

In the third question, students must understand that by changing the unit of measurement for length, the value of the measure, which results from the comparison between the magnitude and the unit of measurement, changes.



Final touches!



1. Describe the situation depicted. What was the female character's intention? Is the situation amusing?
2. What do you think of the association Paige did between the three angles and the reality? For each one of those angles, give other daily life examples.
3. What is the measure of the angles? And what do you call an angle that completes a 90-degree angle? And a 180-degree angle??
4. How can we get a full rotation angle from each of the three angles (using only one of them or by combining, at least, two of them)?
5. Degree Celsius is not the only conventional unit of measurement for temperature in the whole world. Carry out a research to discover what other units are used to measure that magnitude, particularly those used in the United States of America, the country where the strip was produced. Compare them.

Instructions on how to explore the task

Description of the situation

The comic strip that gives rise to this mathematical task belongs to the *FoxTrot* collection, created by the American cartoonist Bill Amend. In this strip, formed by 7 panels, Paige seems to be working on her mathematics homework, trying to represent angles with amplitudes 30° , 90° and 180° . For each of the requests, Paige draws daily life situations that she thinks are associated with each of the angles.

The task

From the task presented it is possible to work on concepts like different types of angles and the different measurement of magnitude units such as the range of angles and temperature. In addition, it is possible to explore the correspondence between degrees Celsius and degrees Fahrenheit.

Evidence shows that Paige is able to identify angles with a certain type of amplitude correctly (at least the ones described). However, the situation is funny because we can see how she associates, creatively, the different meanings of the word “degree”: a unit of measurement for the amplitude of an angle and a unit of measurement for temperature. This association allows her to imagine and to draw, according to the task requested, the angles she associates with real-life contexts. The situation becomes funny because of this unexpected twist. The humour of the task is reinforced by Peter’s remark in which he calls Paige’s illustrations “final touches”.

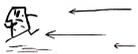
By resorting to two different representations, Paige is using the term "degree" in two distinct ways, that is, she uses, albeit in a seemingly unintentional way, the natural polysemy of the word "degree". The request to draw may be commonly associated with real-life representations and not so much with symbolic representations and that is why Paige feels the need to associate three real-life situations where the temperature is supposedly higher when the angle of the matching drawing also has a higher amplitude.

The third question allows the teacher to recall that angles whose sum is 90° are complementary, while angles whose sum is 180° are supplementary.

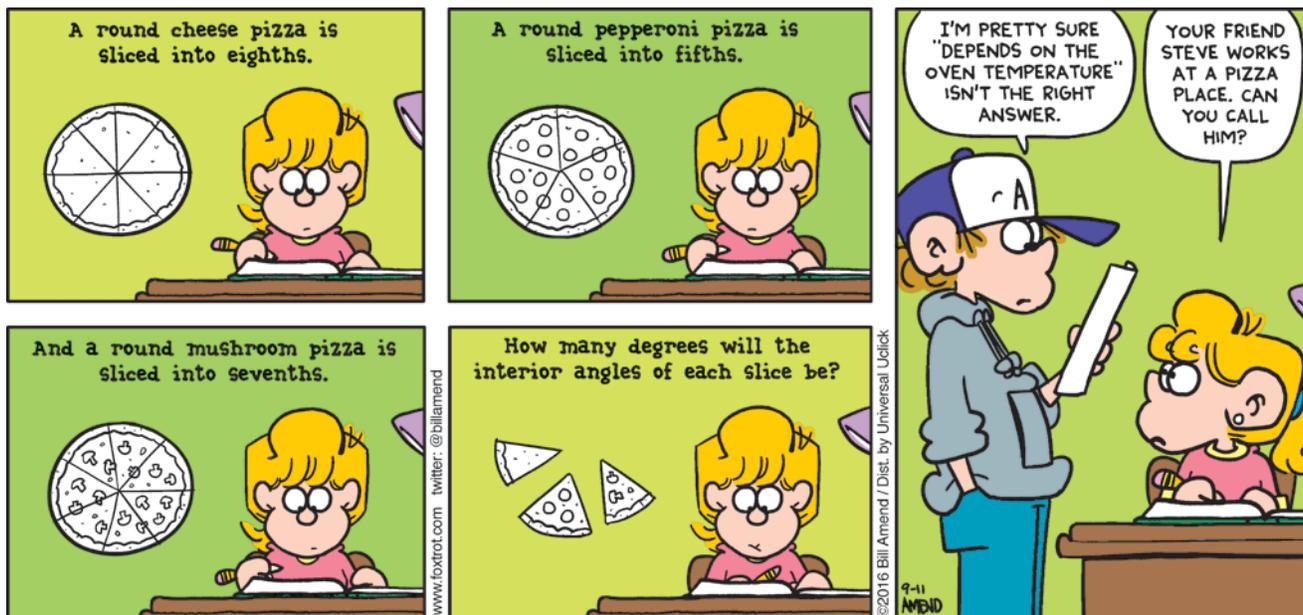
The fourth question deals with the decomposition of the full rotation angle, 360° , using the addition or subtraction of other angles.



The last question allows students to understand that there are different scales to measure the temperature and that the one which is commonly used in the United States of America is the degree Fahrenheit metric measurement, °F, whose relation to degree Celsius, °C, is given by $1^{\circ}C = \frac{1^{\circ}F - 32}{1,8}$. Given the values, students can recognise that, in the strip, the association is being made using degree Fahrenheit.



Pizzas and angles



1. Describe the situation depicted. What do you think of Paige's solution? Do you consider the situation amusing?
2. It is difficult, if not impossible, to make a perfect circular pizza. If you split it into 5, 7 or 8 equal parts, what would the amplitude of each angle be? Does it depend on the size of the pizza? What are your conclusions?
3. If you know the amplitude of the angle of a slice of a pizza divided into 12 equal parts, do you know the amplitudes of the slices of a pizza that is divided into 6 parts or 4 equal parts?

Instructions on how to explore the task

Description of the situation

This mathematical task is based on another *FoxTrot* comic strip by Bill Amend. The strip is made up of two parts. In the first, Paige is solving a school assignment involving a pizza that is being divided into 8, 5 and 7 parts and the amplitude of the central angles of each slice. In the second part, there is a dialogue between Peter and Paige – Peter does not agree with Paige's response according to whom the amplitude of the angles "depends on the temperature of the oven" and Paige suggests that Peter's friend, João, who works in a pizzeria, could help them. The humour lies in the connection that the characters make between the "degrees" (amplitude) of each angle and the "degrees" (temperature) of the pizza oven.

The task

This task makes it possible to work on the concept of central angle, illustrated with questions that involve the division of pizzas. That way, we can also explore the concept of fraction, particularly in situations that would involve the part-whole relationship and operator interpretations.

In the first question, Paige's solution is based on the polysemy of the word "degree", the measure for the amplitude of angles and the measurement unit for temperature magnitude. The humour is caused by this association/confusion.

In the second question, the amplitude corresponding to each slice is determined according to the relationship between each slice (part) and the pizza (whole) and to the fact that the full rotation angle has a 360° amplitude. The reflection around the (im)possibility of dividing, effectively, a pizza into 8, 5 and 7 slices leads to dividers and multipliers relationships between the pairs of number (8.360), (5.360) and (7.360) – In the first case, each slice corresponds to a 45-degree angle; In the second case, to a 72 degree angle. And in the third case, the angle has a measure represented by an infinitely repeating decimal. In the case of the effective division of the pizza into 7 parts, the teacher can suggest a discussion about what should be considered a fair solution.

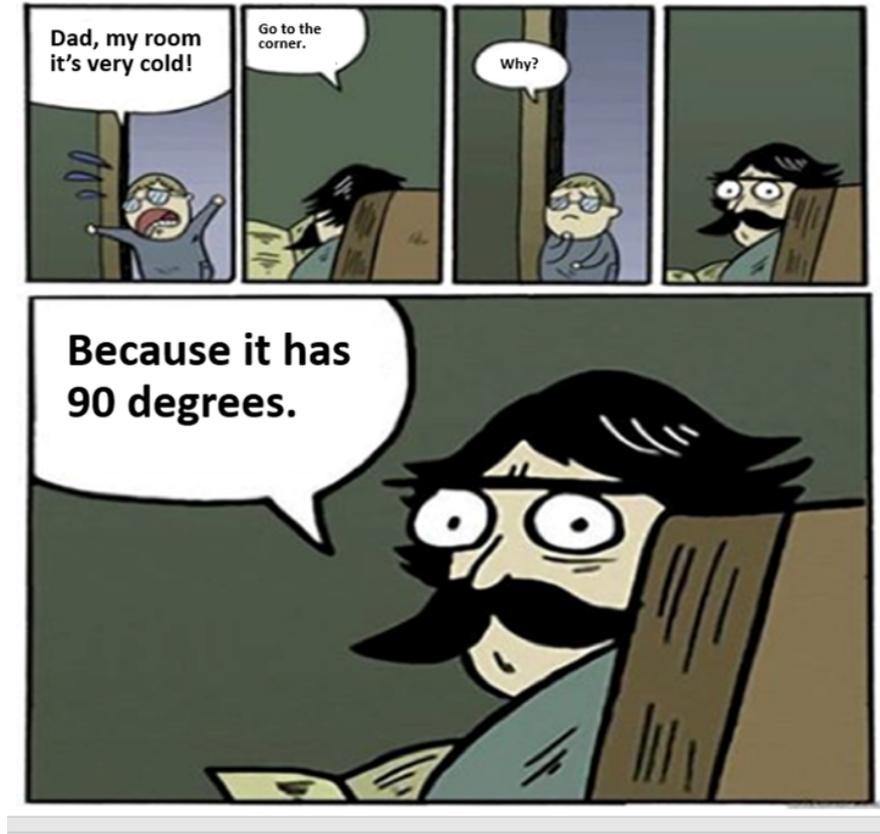
In the third question, students are expected to recognise number 12 as being multiple of 6 and 4 and that the greater the number of parts of pizza is divided into, the smaller the central angle amplitude. That way, we can notice the relationship between the amplitudes of the requested slices, that is to say, between the amplitudes of the central angles:

i) the central angles that correspond to a pizza divided into 6 equal parts have twice the amplitude of the central angles of a pizza divided into 12 equal parts;

ii) the central angles that correspond to a pizza divided into 4 equal parts have three times the amplitude of the central angles of a pizza which had been divided into 12 equal parts.



Degrees and degrees



1. Describe the situation depicted. Do you find it funny?
2. Why does the father send his son to a corner of the room?
3. On a blueprint, the room has a rectangular shape. What is the sum of the amplitude of the interior angles of the room?
4. If a blueprint shows us that the room has the shape of a quadrilateral, what is the sum of the interior angles of that figure?

Instructions on how to explore the task

Description of the situation

This mathematical task uses a comic strip that is available on numerous websites on the internet. In the strip, composed of five panels, the son complains about the cold and his father tells him to move to a corner of the room, since "it is 90 degrees". Like in the "Final touches!" task, the humour of this strip lies in the fact that the author resorted to the polysemy of the word "degree": he used "degree" as a unit of measurement for temperature and as a unit of measurement for the amplitude of angles.

The task

The task designed from this strip allows the teacher and students to focus on concepts like the perpendicularity between two planes, particularly in relation to the right angle that they form and to its amplitude measured using the sexagesimal system, the measurement of temperature in degrees and concave and convex angles.

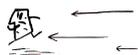
The first question suggests that the student is able to identify, in the situation depicted, the word degree as a unit of measurement for the amplitude of an angle and as a unit of measurement for temperature.

The second question requires the interpretation of the elements that may be involved in the situation pictured and the justification for the use of the word "degree" with both its meanings. Therefore, the following chain of ideas is what we expect students to be able to arrive at:

- a corner composed of different pairs of a perpendicular plane;
- the existence of right angles, that is, angles with a 90° amplitude, in the sexagesimal system;
- "degree" used as a unit of measurement for temperature and the recognition that 90 degrees (in degrees Fahrenheit), corresponds to a very comfortable state in colder days.

The third question uses the situation described to lead the student to conclude that the sum of the amplitude measures of the interior angles of a rectangle equals 360° , since the pairs of consecutive sides are perpendicular.

The last question of the task allows for the generalisation of the findings (made possible following the previous question) to any type of quadrilateral. The decomposition of any quadrilateral into two



triangles (each possessing a sum of the amplitude measures of the interior angles that is equal to 180°) can be a good way of concluding that the sum of the amplitude measures of the interior angles of any quadrilateral equals 360° .



All right!



1. Describe the situation depicted. Do you consider it amusing?
2. The judge and the lawyer are right. What about the defendants? What if we look at them as if they were one?
3. What do we call the type of angles that personify the defendants?

Instructions on how to explore the task

Description of the situation

This mathematical task is based on an illustration of New Zealand's scientist and humourist, Nic D. Kim. The situation describes a trial in which the characters are "angles" – the judge and the defence lawyer are right angles and the defendants are angles with amplitude measures of 42° and 48° . The lawyer addresses the judge using arguments to plead his clients' innocence and claiming that even though they may not meet the desired standards of honesty and that what they did was far from right, together they are able to do the "right" thing. The humour here is brought by the comparison between the word "right" as a synonym for righteous, honest, ethical and the designation of a 90-degree angle.

The task

With this task, you can work concepts like angles (and their descriptions) and complementary angles.

In the first question, students are expected to associate certain angles with the characters and recognise the existence of two acute angles (42° and 48°), the defendants, and two right angles, the judge and the defence lawyer. It is also expected that the association between the characters and mathematical ideas might cause some discussion, notably between judgments (the right way to deal with criminal behaviours), judge (right kind of person), defendants...

In the second question, we want students to make a comparison between the word "right" used as the synonym for sincere, righteous and honest and the designation used to describe an angle with a 90-degree - a right angle. The humour of the situation lies in the personification of the angles and this unexpected association. This question will also allow the teacher to develop the concept of complementary angles.

In the third question, students are expected to associate the defendants with acute angles.



A smaller map



1. What do you think of Lucky Eddie's idea of using a smaller map? Why? Do you consider the situation funny?
2. How did Lucky Eddie find out, just by looking at the map, that they still had 800 miles to go before reaching their destination?
3. If Lucky Eddie had managed a smaller rectangular map of the same region, a quarter the size of the original map, what relationship would there be between the distance, between two places, in both maps? What would the relationship between the scales of both maps be?

Instructions on how to explore the task

Description of the situation

This mathematical task is based on a *Hagar, the Horrible* comic strip, created by the American cartoonist Dik Browne. In this strip, Lucky Eddie and his friend Hagar are observing a map and talking about the distance they still have to cover to get to a certain destination. After realizing that this is quite a considerable distance (800 miles, about 1287 km), Lucky Eddie suggests, naively, that they should have brought a smaller map.

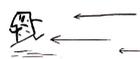
The task

The task presented allows the teacher and students to explore concepts like the notion of scale and, in a more comprehensive way, the concept of direct proportionality. The proposal implies that the relationship between distances should be analysed at two different levels: on the one hand, the relationship between a real distance and the distance represented on a map, according to a given scale and, on the other hand, the relationship between two distances we need to cover to reach two places on two maps with distinct scales and their real corresponding distances.

The first question focuses on the way students will assess Lucky Eddie's reasoning when they see him calculating a distance on the map and suggesting that the best way to shorten that distance would be to get a smaller map. The students reaction to the situation will allow the teacher to assess whether or not they have understood Lucky Eddie's naïve and hilarious logic as he considers that by using a "smaller" map of the same location the actual distances will become smaller.

Question 2 allows for a discussion on the procedures we should use to know real distances using map analysis. There are two kinds of procedures: distances can be determined by means of a visual analysis that requires a comparison with a map unit (for example, a comparison with a distance already travelled), or through the use of a measurement procedure. The measurement of the distance on the map is performed using a measuring instrument, followed by the calculation of the real distance we have to cover, using the scale of that map.

In the first panel, Lucky Eddie determines the distance they still have to travel and a naïve intuition makes him believe that if the map were smaller, the remaining distance would be shorter. This form of reasoning seems to be based on the erroneous conception that the distances on a map can be transposed directly to reality, without taking into account the scale of the map. These considerations help solve the third question of the task that presents an imaginary situation in which a new map, a

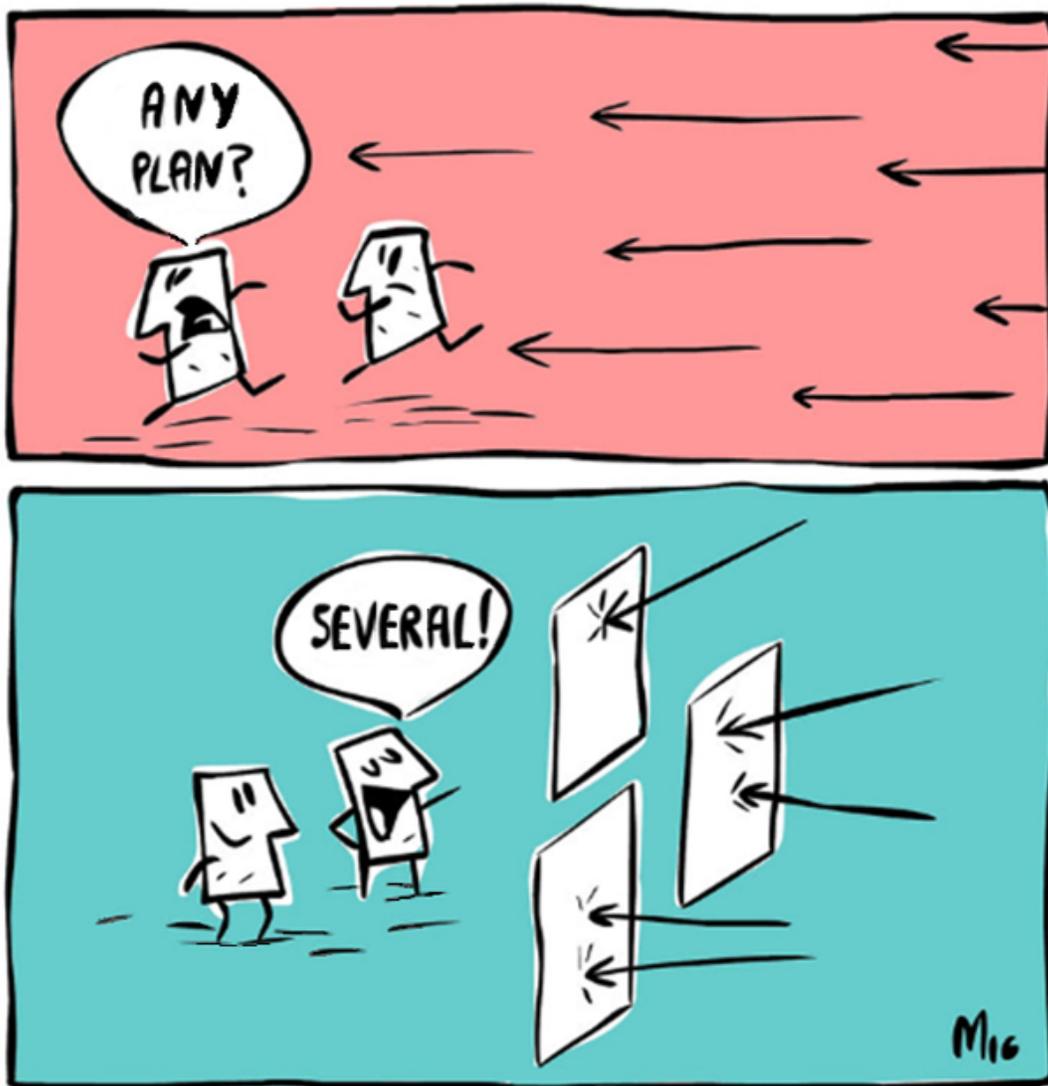


quarter the size of the original area, is considered (a relationship different from those that usually appear on maps and that are built using length measurement).

Based on the relationship between the areas of the two regions on the maps, preferably defined using rectangles, we define the relationship between the segments of lengths on these maps through the property that ensures that if the ratio of the measurements of the areas equals $\frac{1}{4}$, the ratio between the lengths will equal $\frac{1}{2}$.



Plan or planes?!



1. Describe the situation depicted. Do you find the situation amusing?
2. Explain the meaning of the words "plan/plane" suggested by both images. (In Portuguese both words have the same translation "plano")
3. What kind of relationship can be established between planes?
4. Could any of the arrows shot hit the two friends considering the disposition of the different planes? Why not? How many planes would be required to avert the offensive?

Instructions on how to explore the task

Description of the situation

This mathematical task is based on a Marlon Tenório's comic strip, a Brazilian graphic designer⁴. In the situation depicted, two characters find themselves in a critical situation: they are trying to protect themselves against an alleged attack perpetrated with arrows all coming from the same direction. One of them asks if there is an escape or defence plan.

In the second panel, several geometric planes appear and, for that reason, the two friends are now protected against the arrows and therefore against the assault. Each of the panels comes with a warm or cold colour according to the tension of the moment: in the first one a warm colour when the two friends are being attacked and, in the second, a cold colour after the friends found a way to protect themselves.

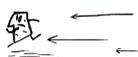
The task

This task allows the study of the concept of plane (representation and properties) and of vector (as an oriented line segment). Based on this strip, students can identify different ways to define a plane and justify their ideas. In addition, it is interesting to discuss the importance of the representation of a plane and the different forms of representation of planes, notably in perspective, like in the strip. For example, it is important for students to understand that although a plane is not a closed sub-region of space, it is usually represented as a closed figure. It can also be interesting to analyse the trajectories of the arrows in the two panels. While in the first panel the trajectory is parallel and rectilinear (when, in reality, they tend to follow a parabolic trajectory), in the second panel the trajectory is no longer parallel, perhaps to reinforce the impression of the impact of the projectiles on the defence planes.

As in other tasks, the humour of this strip focuses on the polysemy of the Portuguese word "plano" (that doesn't exist in English, since "plano" can take on the meaning of plan or plane depending on the context): on the one hand, it can mean *strategy (plan)* and, on the other, it may refer to a *geometric element (plane)*. It is also interesting that the plan the two characters find to defend themselves against the attack is the use of... planes.

The first question focuses on the action. It will also capture the students' attention and interest so they can do their best to explore the meaning of the geometric elements that appear in the strip.

⁴ <http://www.marlontenorio.com/>



In the second question, students must identify the two meanings of the Portuguese word "plano", so they will be able to understand the humour present in the strip.

The goal of the third question is to get students to discuss which relative positions two planes can adopt and whether or not the three planes depicted in the strip may represent the same plane.

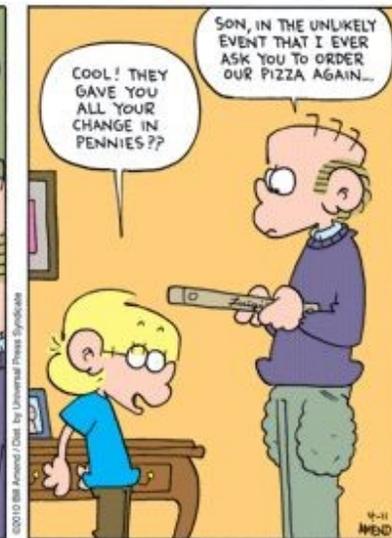
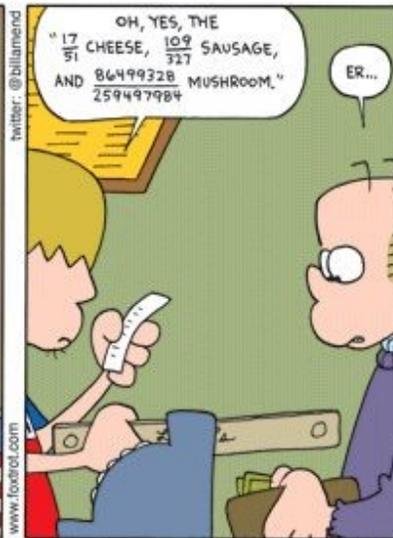
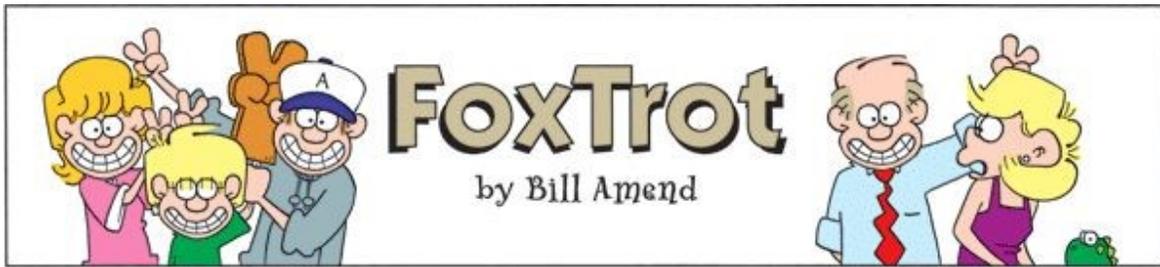
To answer the last question properly, students must conclude that a plan that is not parallel to the arrows' trajectory, at the time of the attack, will be enough to ensure their defence.



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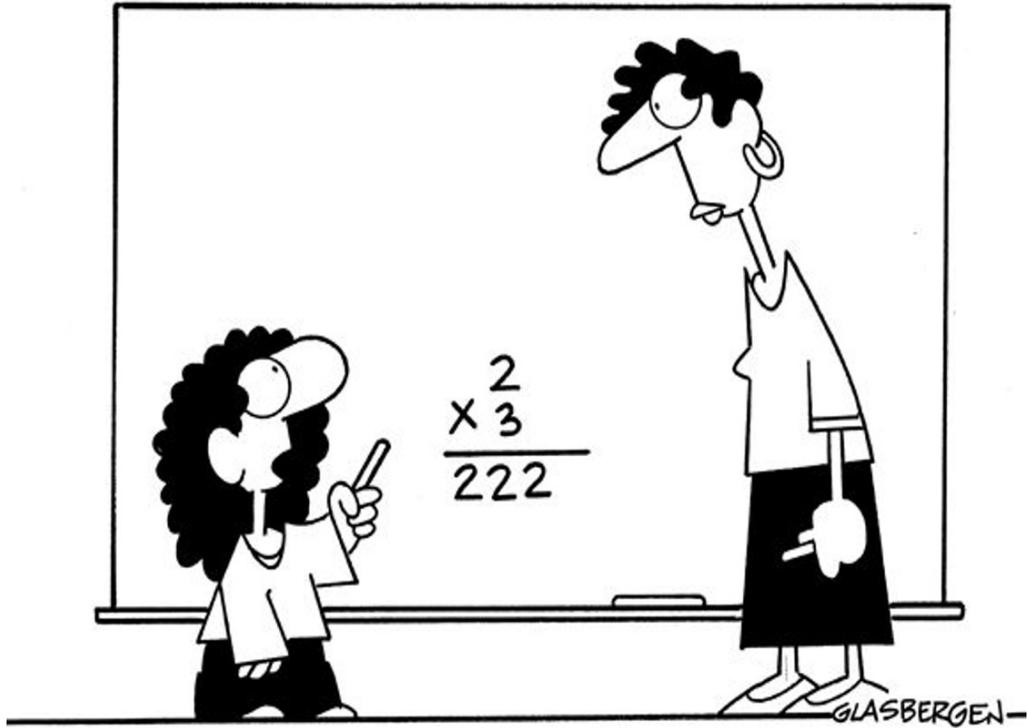
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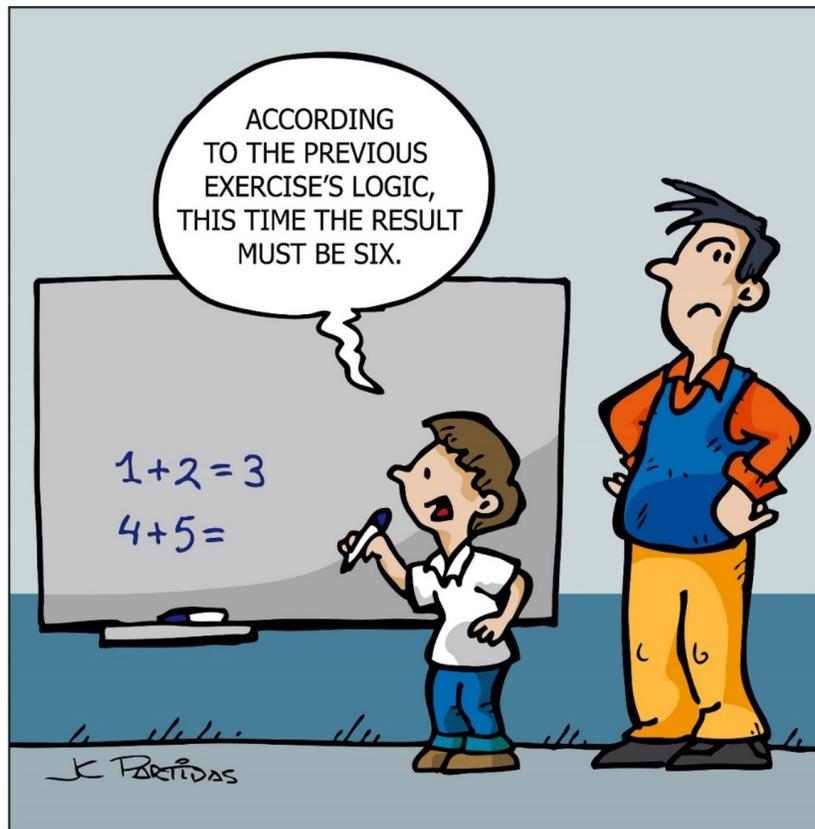
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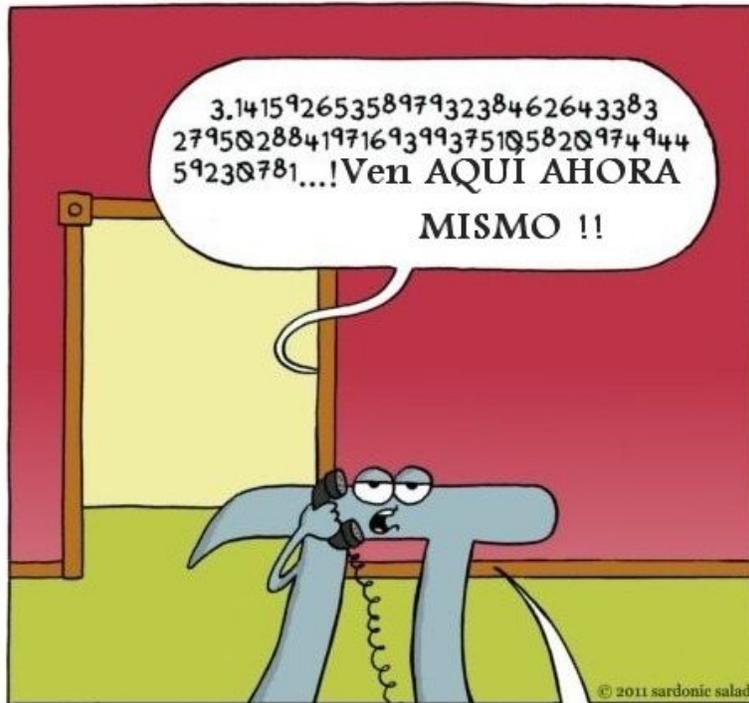
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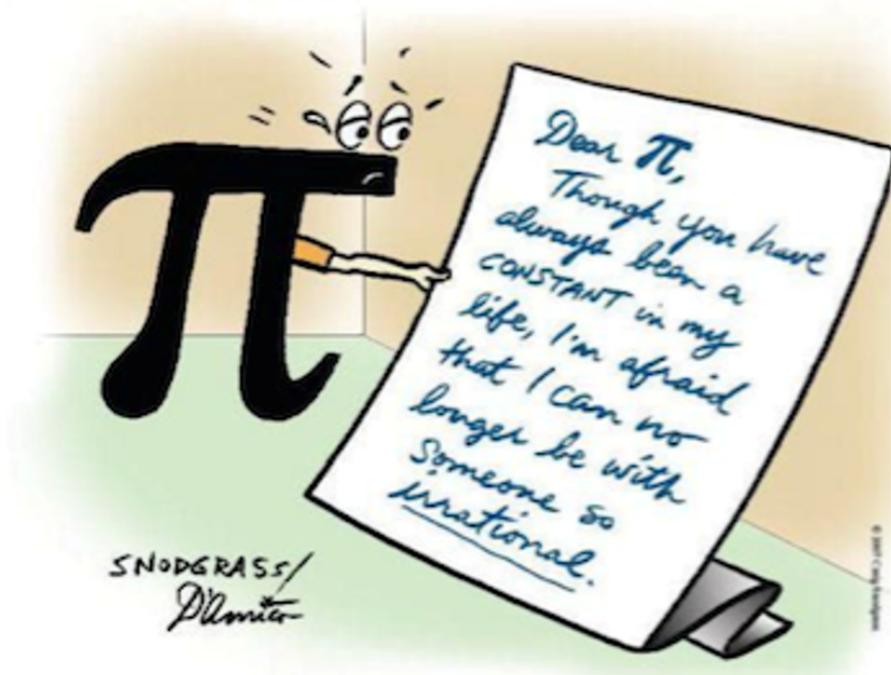


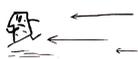
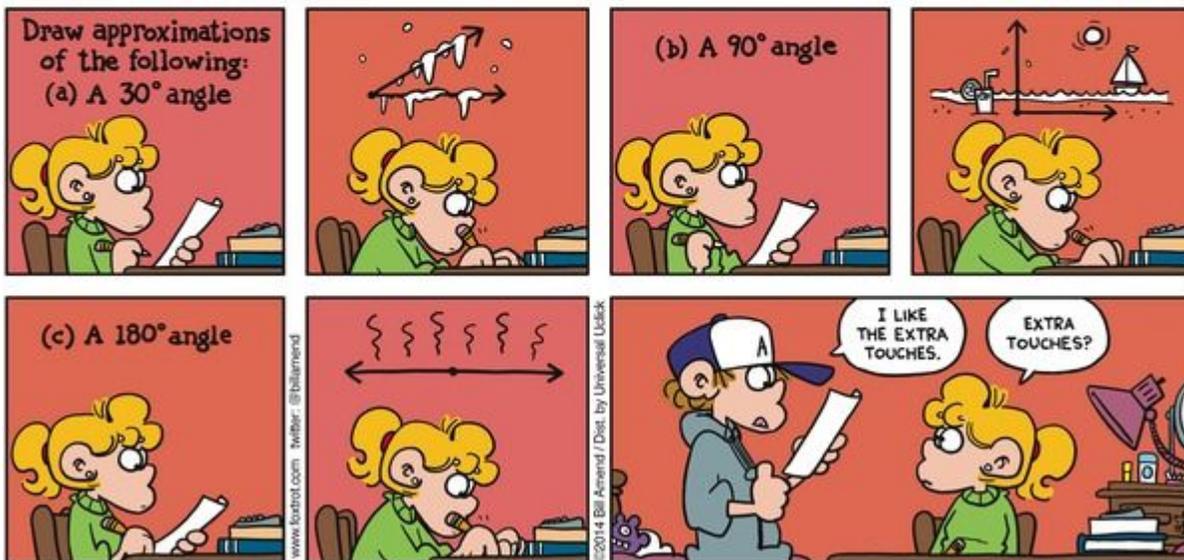
"What do you mean, it's the wrong kind of right?"

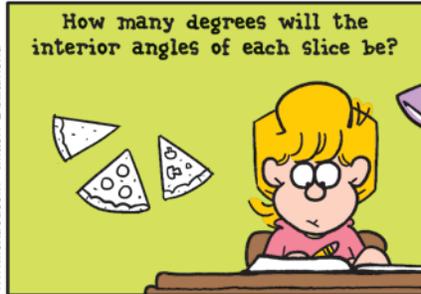
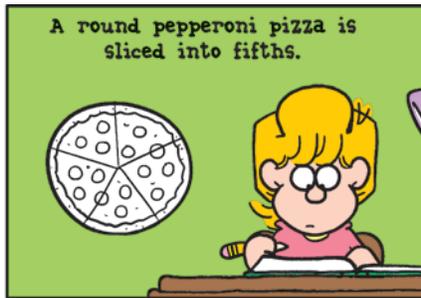
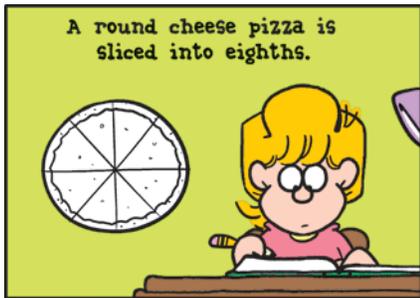




Tengo que ir. Mi madre usa sólo mi nombre completo cuando tengo un problema de verdad...

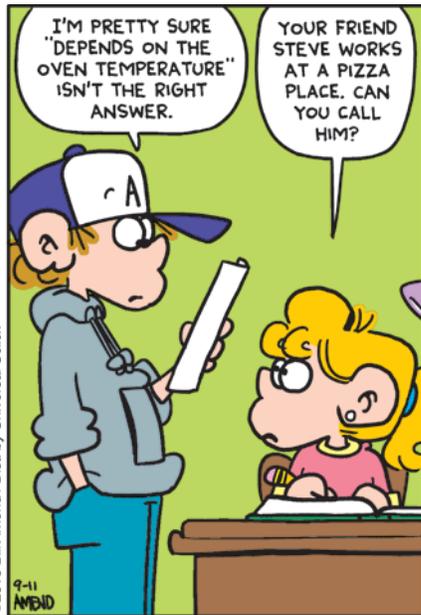






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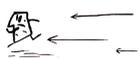
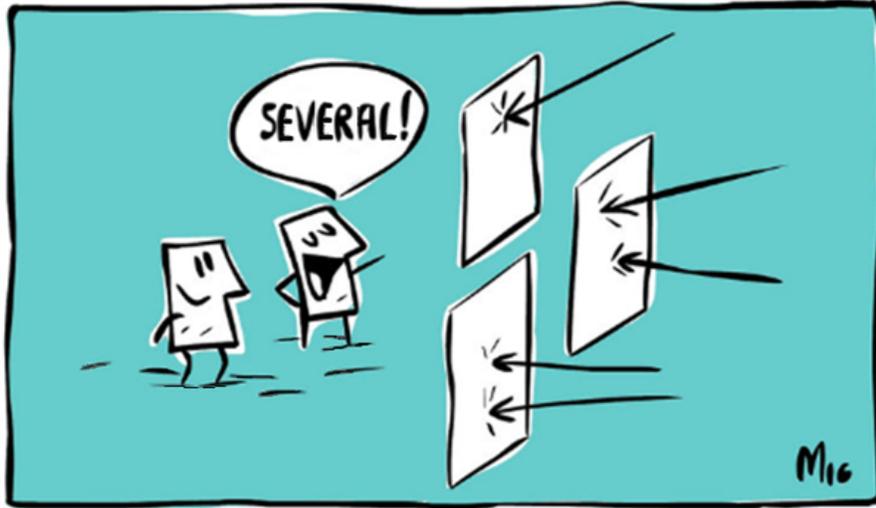
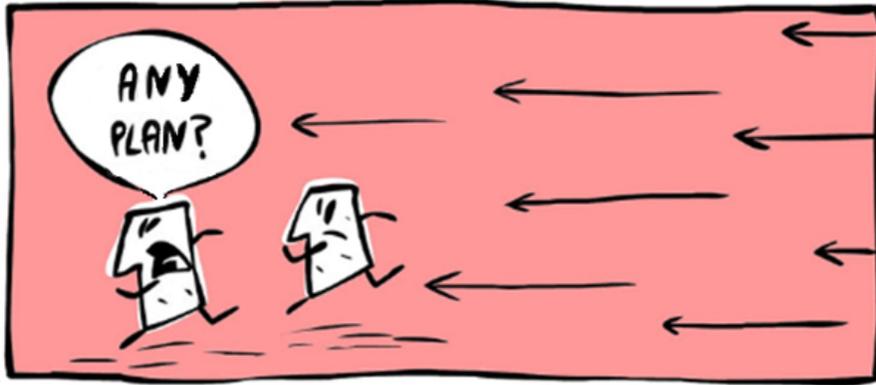


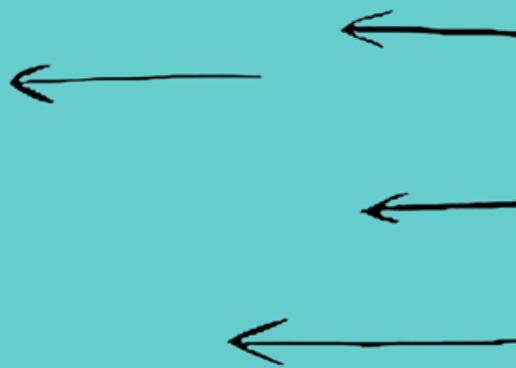


Hagar the Horrible by Dik Browne



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